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Optimum design of double pipe heat exchanger

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Abstract

Heat exchangers are used in industrial processes to recover heat between two process fluids. Although the necessary equations for heat transfer and the pressure drop in a double pipe heat exchanger are available, using these equations the optimization of the system cost is laborious. In this paper the optimal design of the exchanger has been formulated as a geometric programming with a single degree of difficulty. The solution of the problem yields the optimum values of inner pipe diameter, outer pipe diameter and utility flow rate to be used for a double pipe heat exchanger of a given length, when a specified flow rate of process stream is to be treated for a given inlet to outlet temperature.

Keywords: Design; Economy; Geometric programming; Heat exchanger; Optimization

1. Introduction

Heat transfer equipment is defined by the function it fulfills in a process. The objective of any such equipment is to maximize the heat transferred between the two fluids. However, the problem that occurs is that the parameters which increase the heat transfer also increase the pressure drop of the fluid flowing in a pipe which increases the cost of pumping the fluid. Therefore, a design which increases the heat transferred, but simultaneously could keep the pressure drop of the fluid flowing in the pipes to permissible limits, is very necessary. A common problem in industries is to extract maximum heat from a utility stream coming out of a particular process, and to heat a process stream. A solution to extract the maximum heat could have been to increase the heat transfer area or to increase the coolant flow rate but both the solutions increase the cost of pumping so increasing these parameters without pressure drop considerations is not advisable. Traditional design method of heat exchangers involves the consideration of all the design variables with a laborious procedure of trial and error, taking all possible variations into consideration. Though this time consuming procedure can be reduced somewhat by making some reasonable assumptions as described by [6], but still no convenient method has been developed for optimal design of double pipe heat exchangers. In other optimum design methods, such as Lagrange multiplier method, the optimum results are again obtained in a long time by changing one variable at a time and using a trial-error or a graphical method.

In the current literature (for example, [8]), focus is on optimizing the area of the heat exchanger irrespective of the different flow rates of the utility that can be used. Using this pressure drop is not minimized to the fullest extent. This fact can be avoided through the design method discussed in the paper. We have considered the design of a double pipe heat exchanger in which its cost is optimized by considering three main parameters – the

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Nomenclature

Notation

A	constant ($J/m^{0.2}$ s K)
a_p	flow area of inner pipe (m^2)
Re'_a	Reynolds number for pressure drop (Nondimensional)
A_s	surface area for the heat transfer (m^2)
a_a	flow area of annulus (m^2)
B	constant ($J/s^{0.2}$ $m^{0.2}$ $kg^{0.8}$ K)
C	constant (kg $m^{2.75}/s^2$)
C_p	specific heat of fluid flowing in pipe (J/kg K)
d	diameter of inner pipe (m)
D	diameter of outer pipe (m)
D_e	equivalent diameter (m)
E	constant ($m^{2.75}/s^{0.25}$ $kg^{0.75}$)
F	constant (m^2/kg)
f	friction factor (Nondimensional)
g	gravitational acceleration (m/s^2)
G_a	mass velocity of fluid in the annulus (kg/m^2s)
G_p	mass velocity of fluid in the inner pipe (kg/m^2s)
h_i	heat transfer coefficient of inner pipe (J/m^2 s K)
h_o	heat transfer coefficient of annulus (J/m^2 s K)
k_a	thermal conductivity of the fluid flowing in the annulus (J/s mK)
k_p	thermal conductivity of the fluid flowing in the inner pipe (J/s mK)
L	total pipe length (m)
l_{hp}	length of one hairpin (m)
m	mass flow rate of fluid flowing in the inner pipe (kg/s)
Q	heat transfer rate (J/s)
Q^*	required heat transfer rate (J/s)
Re_a	Reynolds number for annulus (–)
Re_p	Reynolds number for inner pipe (–)
T	temperature of the process stream (K)
t	temperature of the utility (K)
U_c	overall clean coefficient (J/m^2 s K)
w	mass flow rate of fluid flowing in the annulus (kg/s)
x	$D^2 - d^2$ (m^2)
y	$D + d$ (m)
ΔT	temperature difference (K)
μ_p	viscosity of fluid flowing in the inner pipe (kg/ms)
μ_a	viscosity of fluid flowing in the annulus (kg/ms)
μ_{wp}	viscosity of fluid in the inner pipe at wall temperature (kg/ms)
μ_{wa}	viscosity of fluid in the annulus at wall temperature (kg/ms)
ρ	fluid mass density (kg/m^3)

Suffixes

1	inlet
2	outlet
a	annulus
io	outer surface of inner pipe
LM	log mean
p	inner pipe

inner and outer diameter of the heat exchanger and the flow rate of the utility. The design of the exchanger has been formulated as a geometric programming with a single degree of difficulty. It is assumed that the flow rate, the inlet and the required outlet temperature of the process fluid and the inlet temperature of the utility are known for the specific design of the exchanger.

2. Analytical considerations

2.1. Equations for heat transfer coefficients for fluids in pipes

Sieder and Tate [3] gave the following equation for both heating and cooling of a number of fluids in pipes:

$$h_i = \frac{0.027}{d} k_p Re_p^{0.8} \left(\frac{C_p \mu_p}{k_p}\right)^{\frac{1}{3}} \left(\frac{\mu_p}{\mu_{wp}}\right)^{0.14} \quad (1)$$

where h_i is the heat transfer coefficient at the inner surface of inner pipe; d the inner pipe diameter (see Fig. 1); k_p the thermal conductivity of the fluid flowing in the inner pipe; C_p the specific heat of fluid flowing in inner pipe; μ_p the viscosity of fluid flowing in the inner pipe; μ_{wp} the viscosity of fluid in the inner pipe at wall temperature; and Re_p the Reynolds number for inner pipe given by

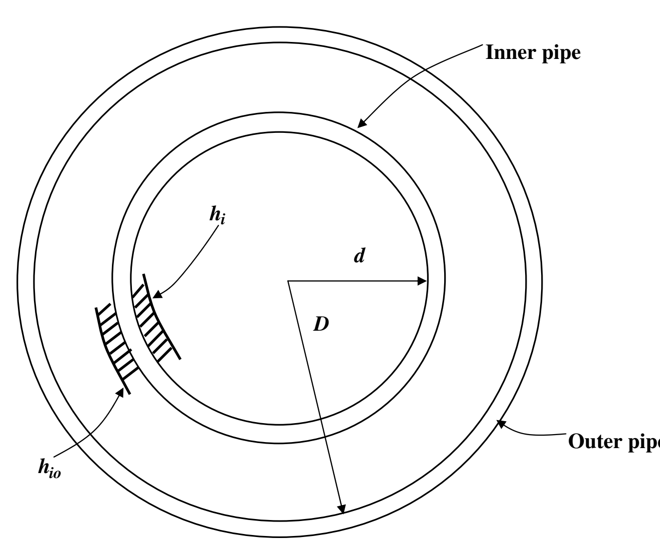

Fig. 1. Cross sectional view of the double pipe heat exchanger.

$$Re_p = \frac{dG_p}{\mu_p} \quad (2)$$

wherein G_p is the mass velocity of fluid in the inner pipe given by

$$G_p = \frac{m}{a_p} \quad (3)$$

wherein m is the mass flow rate of fluid flowing in the inner pipe; and a_p the flow area of inner pipe given by

$$a_p = \frac{\pi}{4} d^2 \quad (4)$$

Combining Eqs. (1)–(4), the following equation is obtained:

$$h_i = \frac{Am^{0.8}}{d^{1.8}} \quad (5)$$

where

$$A = 0.032756 k_p \left(\frac{1}{\mu_p}\right)^{0.8} \left(\frac{C_p \mu_p}{k_p}\right)^{\frac{1}{3}} \left(\frac{\mu_p}{\mu_{wp}}\right)^{0.14} \quad (6)$$

It is assumed that the process stream is pumped in the outer pipe and its flow rate is a known quantity, thus constant for a given problem. However the utility is taken in the inner pipe and its flow rate can be varied so that the required heat transfer is achieved. The thickness of the inner and the outer pipes is also considered negligible with respect to their diameters. Further, as all the other variables are the fluid properties, they are constant for a given fluid. Thus, the heat transfer coefficient of the inner pipe is dependent on its diameter and the flow rate of the utility only for a heat exchanger of given length.

The equivalent diameter D_e for heat transfer in the outer pipe is given by

$$D_e = \frac{D^2 - d^2}{d} \quad (7)$$

where D is the diameter of outer pipe (See Fig. 1). Using [3] equation for calculating the heat transfer coefficient of the outer pipe one gets

$$h_{io} = \frac{0.027}{D_e} k_a Re_a^{0.8} \left(\frac{C_{pa} \mu_a}{k_a}\right)^{\frac{1}{3}} \left(\frac{\mu_a}{\mu_{wa}}\right)^{0.14} \quad (8)$$

where h_{io} is the heat transfer coefficient at the outer surface of the inner pipe; k_a the thermal conductivity of the fluid flowing in the annulus; C_{pa} the specific heat of fluid flowing in annulus; μ_a the viscosity of fluid flowing in the annulus; μ_{wa} the viscosity of fluid in the annulus at outer pipe wall temperature; and Re_a the Reynolds number for the annulus, given by

$$Re_a = \frac{D_e G_a}{\mu_a} \quad (9)$$

where G_a is the mass velocity of fluid in the outer pipe, given by

$$G_a = \frac{w}{a_a} \quad (10)$$

wherein w is the mass flow rate of fluid flowing in the annulus; and a_a the flow area of annulus, given by

$$a_a = \frac{\pi}{4}(D^2 - d^2) \quad (11)$$

Further using Eqs. (7)–(11)

$$h_{io} = \frac{Bd^{0.2}}{x} \quad (12)$$

where

$$x = D^2 - d^2 \quad (13)$$

$$B = 0.032756 k_a \left(\frac{w}{\mu_a}\right)^{0.8} \left(\frac{C_{pa} \mu_a}{k_a}\right)^{\frac{1}{3}} \left(\frac{\mu_a}{\mu_{wa}}\right)^{0.14} \quad (14)$$

Therefore, the heat transfer coefficient of the outer pipe, for a given length of heat exchanger, depends only upon the diameters of both the pipes, other variables being constant for the given process stream.

2.2. Equation for pressure drop in the pipes

The pressure drop allowance in an exchanger is the static fluid pressure which may be expended to drive the fluid through the exchanger. The pressure that is supplied for the circulation of a fluid should overcome frictional losses caused by connecting exchangers in series and the pressure drop in the exchangers itself. The pressure drop in pipes can be computed from the Darcy-Weisbach equation. Therefore, pressure drop Δp_p in the length L of the inner pipe is given by

$$\Delta p_p = \frac{f_p L G_p^2}{2 \rho_p d} \quad (15)$$

where f_p is the friction factor for the inner pipe; and ρ_p the mass density of the inner fluid. Assuming inner pipe to be smooth, $f_p$is given by Blasius equation

$$f_p = \frac{0.316}{Re_p^{0.25}} \quad 2.1 \times 10^3 < Re_p < 10^5 \quad (16)$$

Using (2)–(4), (15) and (16), the pressure drop is

$$\Delta p_p = \frac{0.24113 \mu_p^{0.25} m^{1.75} L}{\rho_p d^{4.75}} \quad (17)$$

The power required by the pump to overcome the pressure drop is

$$P_p = \frac{m}{\rho_p} \Delta p_p \quad (18)$$

Rewriting Eq. (18) using Eq. (17)

$$P_p = \frac{0.24113 \mu_p^{0.25} m^{2.75} L}{\rho_p^2 d^{4.75}} \quad (19)$$

Similarly, the pressure drop in the annulus, Δp_a is

$$\Delta p_a = \frac{f_a G_a^2 L}{2\rho_a D'_e} \tag{20}$$

where ρ_a is the mass density of the outer fluid; D'_e = the equivalent diameter for flow resistance, given by

$$D'_e = D - d \tag{21}$$

f_a is the friction factor for the outer pipe; given by Blasius equation

$$f_a = \frac{0.316}{(Re'_a)^{0.25}} \quad 2.1\times10^3 \leqslant Re'_a \leqslant 10^5 \tag{22}$$

wherein Re'_a = Reynolds number, given by

$$Re'_a = \frac{D'_e G_a}{\mu_a} \tag{23}$$

Combining Eqs. (10) and (11) the mass velocity of fluid in the outer pipe is

$$G_a = \frac{4w}{\pi(D^2 - d^2)} \tag{24}$$

Putting Eqs. (21)–(24) in Eq. (20)

$$\Delta p_a = \frac{0.24113\mu_a^{0.25} y^{1.25} w^{1.75} L}{\rho_a x^3} \tag{25}$$

where

$$y = D + d \tag{26}$$

The power required to pump the fluid against this pressure drop is

$$P_a = \frac{w}{\rho_a}\Delta p_a \tag{27}$$

Rewriting Eq. (27) using Eq. (25)

$$P_a = \frac{0.24113\mu_a^{0.25} y^{1.25} w^{2.75} L}{\rho_a^2 x^3} \tag{28}$$

Adding Eqs. (17) and (25), the overall pressure drop in the heat exchanger is given by

$$\Delta p = \frac{0.24113\mu_p^{0.25} m^{1.75} L}{\rho_p d^{4.75}} + \frac{0.24113\mu_a^{0.25} y^{1.25} w^{1.75} L}{\rho_a x^3} \tag{29}$$

where entrance and exit losses have been neglected.

Adding Eqs. (19) and (28) the total power P, which has to be expended to drive the fluid through the exchanger, is

$$P = \frac{Cm^{2.75}}{d^{4.75}} + \frac{E}{x^3} \tag{30}$$

where

$$C = \frac{0.24113\mu_p^{0.25} L}{\rho_p^2} \tag{31}$$

$$E = \frac{0.24113\mu_a^{0.25} w^{2.75} y^{1.25} L}{\rho_a^2} \tag{32}$$

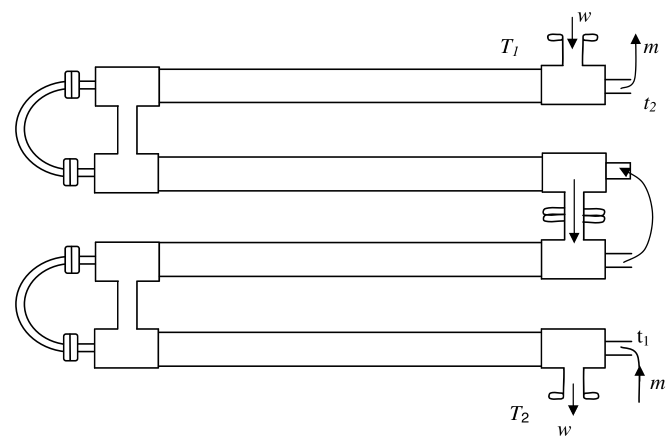

Fig. 2. Four double pipe heat exchangers in series.

2.3. Equation for the heat transferred between the two pipes

Let T_1 and T_2 be the inlet and the required outlet temperature of the process stream and t_1 and t_2 be the inlet and the assumed outlet temperature of the utility. See Fig. 2. The integrated steady state modification of Fourier's general equation is

$$Q = U_c A_s \Delta T_{LM} \tag{33}$$

where Q is the heat transferred between the fluids per unit time; U_c the overall clean coefficient; A_s the surface area for the heat transfer, given by

$$A_s = \pi d L \tag{34}$$

and ΔT_{LM} = log mean temperature difference assuming counterflow

$$\Delta T_{LM} = \frac{(T_1 - t_2) - (T_2 - t_1)}{\ln[(T_1 - t_2)/(T_2 - t_1)]} \tag{35}$$

It is assumed that the thickness of the pipes is negligibly small as compared to the inner diameter of the inner pipe. Overall clean coefficient U_c can be obtained independently of the Fourier equation from the two heat transfer coefficients. Neglecting pipe wall resistance the following equation is obtained:

$$\frac{1}{U_c} = \frac{1}{h_i} + \frac{1}{h_0} \tag{36}$$

Using Eqs. (34)–(36), (33) changes to

$$\frac{1}{Q} = \left(\frac{1}{h_i} + \frac{1}{h_{io}}\right)\frac{1}{\pi d L \Delta T_{LM}} \tag{37}$$

Using Eqs. (5), (12), (37) changes to

$$\frac{1}{Q} = \left(\frac{d^{1.8}}{Am^{0.8}} + \frac{x}{Bd^{0.2}}\right)\frac{1}{\pi d L \Delta T_{LM}} \tag{38}$$

Also, because heat is conserved, the heat lost by the hotter stream is equal to the heat gained by the coolant. Hence,

$$Q = mC_p(t_1 - t_2) \tag{39}$$

and

$$Q = wC_{pa}(T_2 - T_1) \tag{40}$$

3. Objective function

It is desired to operate close to a standard heat transfer rate Q' but on account of increase in pressure we settle for a lesser value Q. Thus, the ratio Q^*/Q being greater than unity, has to be minimized. Similarly, the minimum pumping power required in the process could have been P^*, but one has to settle at a larger pumping power P, so that the heat transferred can be increased; and as a result the ratio P/P^* has to be decreased. Also it would have been preferred had the utility pumped was minimum given by m^*, but again to increase the heat transferred more utility is used in the process (a similar approach for optimizing the performance of a furnace can be found in [2]). Therefore, the objective function F, which is to be minimized, is selected as

$$F_1 = \frac{Q'C_1}{Q} + \frac{P_pC_2' + P_aC_3'}{P'} + \frac{m}{m'}C_3 \tag{41}$$

where C_1, C_2', C_3' and C_3 are the coefficients having monetary units. The first term on the Right hand side is the cost related with the external heat provided by the utility. The second term is related with the cost required to provide the power to pump the fluids. The last term is the cost of the utility used.

Also $C_2' \approx C_3'$. Therefore writing these two costs as C_2, Eq. (41) becomes

$$F_1 = \frac{Q'}{Q}C_1 + \frac{P}{P'}C_2 + \frac{m}{m'}C_3 \tag{42}$$

It is assumed that in long run the capital cost of the heat exchanger employed is not that significant as compared to the operating cost. Therefore to calculate the minimum cost of the heat exchanger the focus will be laid on minimizing this function.

Using Eqs. (30), (38), (42) changes to

$$F_1 = A'd^{0.8}m^{-0.8} + B'xd^{-1.2} + C'm^{2.75}d^{-4.75} + E'x^{-3} + F'm \tag{43}$$

where

$$A' = \frac{Q'C_1}{\pi L\Delta T_{LM}A} \tag{44}$$

$$B' = \frac{Q'C_1}{\pi L\Delta T_{LM}B} \tag{45}$$

$$C' = \frac{CC_2}{P'} \tag{46}$$

$$E' = \frac{EC_2}{P'} \tag{47}$$

$$F' = \frac{C_3}{m'} \tag{48}$$

Eq. (43) is in the form of a posynomial (positive polynomial). Thus the minimization of Eq. (43) boils down to a geometric programming problem with a single degree of difficulty (see [1]). The contributions of various terms of Eq. (43) are defined by the weights w_1, w_2, w_3, w_4 and w_5 (see [5]), given by

$$w_1 = A'd^{0.8}m^{-0.8}F_1^{-1} \tag{49}$$

$$w_2 = B'xd^{-1.2}F_1^{-1} \tag{50}$$

$$w_3 = C'm^{2.75}d^{-4.75}F_1^{-1} \tag{51}$$

$$w_4 = E'x^{-3}F_1^{-1} \tag{52}$$

$$w_5 = F'mF_1^{-1} \tag{53}$$

The dual objective function F_2 of Eq. (43) is written as

$$F_2 = \left(\frac{A'd^{0.8}}{m^{0.8}w_1}\right)^{w_1}\left(\frac{B'x}{d^{1.2}w_2}\right)^{w_2}\left(\frac{C'm^{2.75}}{d^{4.75}w_3}\right)^{w_3}\left(\frac{E'}{x^3w_4}\right)^{w_4}\left(\frac{F'm}{w_5}\right)^{w_5} \tag{54}$$

The orthogonality condition of Eq. (43) for d, x and w are

$$d\!: 0.8w_1^* - 1.2w_2^* - 4.75w_3^* = 0 \tag{55}$$

$$x\!: w_2^* - 3w_4^* = 0 \tag{56}$$

$$w\!: -0.8w_1^* + 2.75w_3^* + w_5^* = 0 \tag{57}$$

whereas the normality condition of Eq. (43) is

$$w_1^* + w_2^* + w_3^* + w_4^* + w_5^* = 1 \tag{58}$$

where * corresponds to optimality. Solving Eqs. (55)–(58) for w_1^*, w_2^*, w_3^* and w_4^*, one gets

$$w_1^* = -3.9655w_5^* + 1.5517 \tag{59}$$

$$w_2^* = 3.3621w_5^* - 0.7524 \tag{60}$$

$$w_3^* = -1.5172w_5^* + 0.4514 \tag{61}$$

$$w_4^* = 1.1207w_5^* - 0.2508 \tag{62}$$

Substituting Eqs. (55)–(58) in Eq. (54), and simplifying, the optimal dual F_2^* is

$$F_2^* = \frac{0.2503(w_5^*-0.2238)^{1.0032}}{(0.3913-w_5^*)^{1.5517}(0.2975-w_5^*)^{0.4514}}\frac{A'^{1.5517}C'^{0.4514}}{B'^{0.7524}E'^{0.2508}} \times\left[\frac{6.6260(0.3913-w_5^*)^{3.9655}(0.2975-w_5^*)^{1.5172}}{(w_5^*-0.2238)^{4.4828}w_5^*}\frac{B'^{3.3621}E'^{1.1207}F'}{A'^{3.9655}C'^{1.5172}}\right]^{w_5^*} \tag{63}$$

The dual function given by Eq. (63) depends on the optimal weight w_5^*, which can be obtained by differentiating Eq. (63) with respect to w_5^*, equating it to zero, and simplifying. Considering the complexity of Eq. (63), it can be seen that this approach is prohibitive. Alternatively, equating the factor having exponent w_5^*, on right hand side of Eq. (63) to unity the optimality condition is obtained as ([4])

$$\frac{6.6260(0.3913-w_5^*)^{3.9655}(0.2975-w_5^*)^{1.5172}}{(w_5^*-0.2238)^{4.4828}w_5^*}\frac{B'^{3.3621}E'^{1.1207}F'}{A'^{3.9655}C'^{1.5172}} = 1 \tag{64}$$

Eq. (64) is written as

$$M = \frac{6.6260(0.3913-w_5^*)^{3.9655}(0.2975-w_5^*)^{1.5172}}{(w_5^*-0.2238)^{4.4828}w_5^*} \tag{65}$$

where M is a parameter given by

$$M = \frac{A'^{3.9655} C'^{1.5172}}{B'^{3.3621} E'^{1.1207} F'} \tag{66}$$

Eq. (66) is an implicit equation in w_5^*. For practical ranges of M it can be fitted to the following explicit form in w_5^*:

$$w_5^* = 0.2975 - \frac{0.0737}{1 + 4.2M^{-0.277}} \tag{67}$$

Using Eqs. (63) and (67), the maximum of the dual F_2^* is obtained. As the maximum of the dual is equal to the minimum of the primal, i.e., $F_1^* = F_2^*$, the optimal cost is obtained as

$$F_1^* = \frac{0.2503(w_5^* - 0.2238)^{1.0032}}{(0.3913 - w_5^*)^{1.5517}(0.2975 - w_5^*)^{0.4514}} \frac{A'^{1.5517} C'^{0.4514}}{B'^{0.7524} E'^{0.2508}} \tag{68}$$

A perusal of Eq. (65) reveals that as M varies between 0 and ∞, w_5^* varies between 0.2975 and 0.2238. Thus, w_5^* has a narrow range $0.2238 \leqslant w_5^* \leqslant 0.2975$. Combining Eqs. (53) and (68) the optimal flow rate m^* is

$$m^* = \frac{0.2503(w_5^* - 0.2238)^{1.0032} w_5^*}{(0.3913 - w_5^*)^{1.5517}(0.2975 - w_5^*)^{0.4514}} \frac{A'^{1.5517} C'^{0.4514}}{B'^{0.7524} E'^{0.2508} F'} \tag{69}$$

Similarly, using Eqs. (50, 52, 60, 62 and 69), the following equations are obtained:

$$x^* = \frac{1.5275(0.3913 - w_5^*)^{0.5172}(0.2975 - w_5^*)^{0.1505}}{(w_5^* - 0.2238)^{0.6677}} \frac{B'^{0.2508} E'^{0.4169}}{A'^{0.5172} C'^{0.1505}} \tag{70}$$

$$d^* = \frac{0.2481(w_5^* - 0.2238)^{2.2572} w_5^*}{(0.3913 - w_5^*)^{2.2413}(0.2975 - w_5^*)^{1.0156}} \frac{A'^{2.2413} C'^{1.0156}}{B'^{1.6929} E'^{0.5643} F'} \tag{71}$$

The proceeding development is based on the assumption that y, T_2, and t_2 are constants. The variation in these variables can be considered by an iterative procedure. The successive iterative values of the temperatures T_2 and t_2 are found to be oscillating if the usual iteration procedure is carried out. Therefore, the following scheme is used so that the number of steps needed to obtain the result can be decreased.

1. Assume a value of D, d, m, T_2.
2. Find t_2 by using (39) and (40) with $t_2 = t_1 - \frac{wC_{pa}(T_2 - T_1)}{mC_p}$.
3. Find y using Eq. (26).
4. Find A', B', C', E', F' using Eqs. (44)–(48).
5. Find M using Eq. (66).
6. Find w_5^* using Eq. (67).
7. Find m^* using Eq. (69).
8. Find x^* using Eq. (70).
9. Find d^* using Eq. (71).
10. Find Q by using Eq. (38).
11. Find D using Eq. (13).
12. Find T_2 by using (39) with $T_2 = \frac{Q}{wC_{pa}} + T_1$.
13. Find t_2 by using (40) with $t_2 = t_1 - \frac{Q}{m^* C_p}$.
14. Find new $T_2 =$ (new T_2 + old T_2)/2.
15. Find new$t_2 =$ (new t_2 + old t_2)/2.
16. Repeat steps 3–15 till two successive y, T_2 and t_2 values are close.
17. Reduce d^* and D^* to the nearest commercially available size.
18. Find the actual value of y using Eq. (26).
19. Find the actual value of x^* using Eq. (13).
20. Find the corrected values of A', B', C', E', F' using Eqs. (44)–(48).
21. Find the actual value of m^* by using Eq. (69).
22. Find the actual value of Q by using Eq. (38).
23. Find actual T_2 by using (39) with $T_2 = \frac{Q}{wC_{p}a} + T_1$.
24. Find actual t_2 by using (40) with $t_2 = t_1 - \frac{Q}{m^* C_p}$.
25. Find P by using (30).
26. Find F_1 by using (43).
27. Find the actual values of$w_1^*, w_2^*, w_3^*, w_4^*$ and w_5^* using Eqs. (49)–(53).

4. Design example

It is desired to heat 1.512 kg/s of cold benzene from 300 K to the maximum temperature possible by using hot toluene which is available at 344 K. The fluid properties are: For benzene $\rho_a = 880$ kg/m^3; $\mu_a = 0.00050$ kg/ms; $C_{pa} = 1778$ J/kg K; $k_{pa} = 0.1574$ J/s mK; and for toluene $\rho_p = 870$ kg/m^3; $\mu_p = 0.00041$ kg/ms; $C_p = 1817$ J/kg K; and $k_p = 0.1471$ J/s mK. The fluid properties are assumed to be constant with respect to temperature change ensuring $\mu_p = \mu_{wp}$ and $\mu_a = \mu_{wa}$. For this operation three double pipe heat exchangers of 6.096 m length are available. It is assumed that the cost related to heat flow rate is 5.1$, that to pumping is 1.09$ and with the utility in this case is 1.4$. (i.e. $C_1 = 5.1$\$, $C_2 = 1.09$\$ and $C_3 = 1.4$)$, (these costs may differ from industry to industry). The standard heat transfer rate, Q^*, is 94092 J/s and it is required that the process operates close to this. The minimum pumping power, P^*, is 117 W. Find out the optimum values of inner pipe diameter, outer pipe diameter and coolant flow rate to be used for the heat exchangers which are to be connected in series.

The process stream, i.e. cold benzene in this case, is taken in the outer pipe and the utility stream, hot toluene, is kept in the inner pipe. For starting the algorithm, the initial values of outer diameter of inner and outer pipes were assumed to be 0.042 m and 0.06 m, respectively. Therefore, the initial value of y is $d + D = 0.102$ m. The initial flow rate of the utility was assumed to be 3.6989 kg/s. The initial outlet temperature of the process stream is assumed 335 K and the iterations were carried out till two successive values of y, T_2 and t_2 were obtained. These iterations have been shown in Table 1. The values of the inner and outer diameters of the pipes finally obtained are 0.027 and 0.049, respectively. Thus the commercially available pipes of inner

Table 1
Design iterations: physical variables

Iteration number (1)	w_5^* (2)	d(m) (3)	m(kg/s) (4)	Q (J/s) (5)	D (m) (6)	y (m) (7)	T_2 (K) (8)	t_2 (K) (9)
0	–	0.042	3.6989	–	0.06	0.102	335	330
1	0.2891	0.0259	1.2729	54106	0.0502	0.1989	327.5632	325.3028
2	0.2884	0.0259	1.1562	66242	0.0486	0.2488	326.1019	318.8854
3	0.2882	0.0269	1.2496	63225	0.0486	0.2683	324.8101	317.5192
4	0.2882	0.0268	1.2508	62814	0.0487	0.2754	324.0877	316.9403
5	0.2882	0.0268	1.2487	62881	0.0487	0.2779	323.7391	316.6127
6	0.2882	0.0268	1.2492	62872	0.0487	0.2787	323.5631	316.4562
7	0.2882	0.0268	1.2492	62868	0.0487	0.2787	323.4742	316.3798

diameter 0.035 m and outside diameter of 0.0525 m can be provided (The commercially available diameters are adopted from fps system). The tube diameters are discontinuous variables due to standardization. After adopting the commercially available diameters the actual value of the weights were found and the optimum value of the mass flow rate was calculated by Eq. (69). The optimum values were also checked for different pairs of the inner and outer diameters. The values of the diameters taken were $D_1 = 0.052$ m and $D_2 = 0.041$ m and $d_1 = 0.035$ m, and $d_2 = 0.027$ m. Therefore, four combinations were tried with varying the mass flow rates. The minimum cost comes out to be for $D = 0.052$ m and $d = 0.035$ m at $m = 1.2492$ kg/s. Therefore the coolant flow rate to be pumped is 1.25 kg/s. The final value of the weights $w_1^*, w_2^*, w_3^*, w_4^*$ and w_5^* comes out to be 0.4088, 0.2062, 0.0141, 0.0722 and 0.2882. Using Eq. (42) it can be seen that the contribution of the heating flow rate in cost is around 60%.

5. Salient points

The outer diameter of the heat exchanger is around 1.8 times the inner diameter. The total heat transfer area to be provided is around 6.159 m^2 which suggests that double pipe exchangers would be the best type of heat exchanger equipment for the process. The fact, that the optimal weight w_1^* is greater than w_2^*, tells that the heat transfer coefficient of the inner pipe is lower than that of the outer pipe. The optimal weight w_4^* is around five times w_3^*, which indicates that for optimal conditions the power needed to make the fluid flow in the annulus is quite larger than that needed in the inner pipe (though the range of w_3^* is larger than that of w_4^* but the optimal value obtained is five times less). Also w_1^* and w_2^* are greater than the optimal weights for pressure drop which simply indicates that the cost due to heat transfer is more than the pumping cost and the cost of the utility used. The outlet temperature of the process stream is around 323 K which is well below the approachable temperature indicating the practicality of the solution. The efficiency of the exchanger as per the definition given by [7] is around 63.6% which is reasonably high.

For a feasible solution to the problem the weights can lie anywhere between the ranges: $0.3620 \leqslant w_1^* \leqslant 0.6642; 0 \leqslant w_2^* \leqslant 0.2478; 0 \leqslant w_3^* \leqslant 0.1118; 0 \leqslant w_4^* \leqslant 0.0826$; and $0.2238 \leqslant w_5^* \leqslant 0.2975$. These ranges indicate contributions of different terms in the optimal objective function. Most dominant term is the first term. This suggests that the heat transfer rate has the maximum importance. Thus the solution confirms maximum heat flow rate. The least significant term is the fourth term. It can as well be removed to yield a simple solution.

6. Conclusion

It has been possible to formulate the optimal design of heat exchanger as a geometric programming problem having single degree of difficulty. Since the optimal design minimizes the weighted sum of the heat transfer cost, the pumping cost and the cost of the utility used, by changing the weights one can achieve higher heat transfer by appropriate changes.

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